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Hall effect on thermosolutal instability in a Maxwellian viscoelastic fluid in porous medium

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THERMOSOLUTAL instability in a Maxwellian viscoelastic fluid in porous medium is studied to include the effect of Hall current. For stationary convection, the Maxwellian viscoelastic fluid behaves like an ordinary Newtonian fluid and stable solute gradient is found to have stabilizing effect, whereas Hall currents and medium permeability are found to have destabilizing effects on the system. The sufficient conditions for the non-existence of overstability are also obtained.

1. Introduction

THE ONSET OF CONVECTION in Newtonian fluids heated from below, under varying assumptions of hydrodynamics and hydromagnetics, has been treated by CHANDRASEKHAR [1]. The effect of Hall currents on the thermal instability of a horizontal layer of conducting fluid has been studied by GUPTA [2]. VERONIS [3] has investigated the thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The heat and solute being two diffusing components, thermosolutal convection is the general term dealing with such phenomena.

A macroscopic equation which describes incompressible flow of a Newtonian fluid of viscosity μ through a macroscopically homogeneous and isotropic porous medium of permeability k_1 is the well known Darcy's equation. The usual viscous term in the equations of fluid motion is replaced by the resistance term $-(\mu/k_1)\mathbf{v}$, where $\mathbf{v}$ is the filter velocity of the fluid.

BHATIA and STEINER [4] have studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation and have found that the rotation has a destabilizing effect, in contrast to the stabilizing effect on Newtonian fluid. BHATIA and STEINER [5] have also considered the thermal instability of a Maxwell fluid in hydromagnetics and have found that the magnetic field has stabilizing effect on viscoelastic fluid, just as in case of Newtonian fluid.

The Hall effect is likely to be important in many geophysical situations like Earth's molten core as well as in flows of laboratory plasma. SHERMAN and SUTTON [6] have considered the effect of Hall current on the efficiency of a magneto-fluid-dynamic generator. UBEROI and DEVANATHAN [7] have investigated the effects of Hall phenomenon on the propagation of small amplitude waves taking compressibility into account. As the Hall current, solute gradient and viscoelastic effects are likely to be important in geophysical situations, a reconsideration of the thermal convection effects occurring in porous medium including these effects is certainly called for and is the object of the present paper.

2. Perturbation equations

Here we consider an infinite horizontal layer of a Maxwellian viscoelastic fluid of depth d in a porous medium, heated and soluted from below and acted on by gravity force $\mathbf{g}(0,0,-g)$ and magnetic field $\mathbf{H}(0,0,H)$. The Maxwell's viscoelastic fluid is described by the constitutive relations

$$\begin{aligned} T_{ij} &= -p\delta_{ij} + \tau_{ij}\,, \\ \left(1+\lambda\frac{d}{dt}\right)\tau_{ij} &= z\mu e_{ij}\,, \\ e_{ij} &= \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j}+\frac{\partial q_j}{\partial x_i}\right), \end{aligned} \tag{2.1}$$

where T_{ij}, $\tau_{ij}(=\boldsymbol{\tau})$, $e_{ij}(=\mathbf{e})$, δ_{ij}, p, q_i, x_i, μ and λ denote respectively the stress tensor, shear stress tensor, rate-of-strain tensor, Kronecker delta, scalar pressure, velocity, position vector, viscosity and stress relaxation time. d/dt is the convective derivative.

When the fluid slowly percolates through the pores of the rock, the gross effect is represented by the usual Darcy's law. As a consequence, the resistance term $-(\mu/k_1)\mathbf{v}$ will replace the usual viscous term in the equation of motion. Here k_1 is the permeability of the medium and $\mathbf{v}$ is the filter velocity of the fluid.

The equations of motion, continuity and heat conduction for a viscous, incompresible fluid heated from below (Chandrasekhar [1], pp. 11–16) are

$$\varrho\frac{d\mathbf{q}}{dt} = \varrho\mathbf{X} - \nabla p + \operatorname{div}\boldsymbol{\tau}\,, \tag{2.2}$$

$$\nabla\cdot\mathbf{q} = 0, \tag{2.3}$$

$$\varrho\frac{d}{dt}(c_v T) = \frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right), \tag{2.4}$$

where p, ϱ, T, $\mathbf{q}$ and $\mathbf{X}$ denote respectively the fluid pressure, density, temperature, velocity and the external force acting on the fluid. k and c_v stand for the thermal conductivity and the specific heat at constant volume. The viscous dissipation term, being very small in magnitude, has not been included in (2.4). Since external forces are of non-electromagnetic origin (gravity) and of electromagnetic origin (Lorentz force per unit volume), equation of motion (2.2) may be rewritten as

$$\varrho\frac{d\mathbf{q}}{dt} = -\nabla p + \varrho\mathbf{g} + \frac{\mu_e}{4\pi}(\nabla\times\mathbf{H})\times\mathbf{H} + \operatorname{div}\boldsymbol{\tau}. \tag{2.5}$$

Using the constitutive relations (2.1) for the Maxwellian viscoelastic fluid and also using the fact that when fluid flows through a porous medium, the gross effect is represented by Darcy's law, the equations of motion and continuity for a Maxwellian viscoelastic fluid through porous medium become

$$\frac{\varrho}{\varepsilon}\left(1+\lambda\frac{d}{dt}\right)\frac{d\mathbf{v}}{dt}=\left(1+\lambda\frac{d}{dt}\right)\left[-\nabla p+\varrho\mathbf{g}+\frac{\mu_e}{4\pi}(\nabla\times\mathbf{H})\times\mathbf{H}\right]-\frac{\varrho\nu}{k_1}\mathbf{v}, \tag{2.6}$$

$$\nabla\cdot\mathbf{v}=0, \tag{2.7}$$

where $\mathbf{v}$ is the filter velocity, ε is medium porosity and k_1 is the medium permeability. $\nu(=\mu/\varrho)$ and μ_e stand for kinematic viscosity and magnetic permeability. The fluid velocity $\mathbf{q}$ and the Darcian (filter) velocity $\mathbf{v}$ are connected by the relation $\mathbf{q}=\mathbf{v}/\varepsilon$.

When the fluid flows through a porous medium, the equation of heat conduction (JOSEPH [8],pp. 53–55) is

$$[\varrho c_f\phi+\varrho_s c_s(1-\phi)]\frac{\partial T}{\partial t}+\varrho c_f(\mathbf{v}\cdot\nabla)=k\nabla^2T. \tag{2.8}$$

An analogous solute concentration equation is

$$\left[\varrho c'_f\phi+\varrho_s c'_s(1-\phi)\right]\frac{\partial C}{\partial t}+\varrho c'_f(\mathbf{v}\cdot\nabla)C=k'\nabla^2C. \tag{2.9}$$

Using generalized Ohm's law to take account of the Hall current

$$\mathbf{j}=\sigma(\mathbf{E}+\mathbf{v}\times\mathbf{B})-\frac{c}{Ne}\mathbf{j}\times\mathbf{H}, \tag{2.10}$$

and eliminating $\mathbf{E}$, $\mathbf{j}$ etc., the Maxwell's equations in terms of magnetic field become

$$\frac{\partial\mathbf{H}}{\partial t}=\frac{1}{\varepsilon}\nabla\times(\mathbf{v}\times\mathbf{H})+\eta\nabla^2\mathbf{H}-\frac{c}{4\pi Ne}\nabla\times[(\nabla\times\mathbf{H})\times\mathbf{H}], \tag{2.11}$$

$$\nabla\cdot\mathbf{H}=0. \tag{2.12}$$

Initially

$$\mathbf{v}=(0,0,0),\qquad \varrho=\varrho(z),\qquad p=p(z),$$

$$T=T(z),\qquad C=C(z)\qquad\text{and}\qquad \mathbf{H}=(0,0,H).$$

Let $\delta\varrho$, δp, θ, γ, $\mathbf{h}(h_x,h_y,h_z)$ and $\mathbf{v}(u,v,w)$ denote respectively the perturbations in density ϱ, pressure p, temperature T, solute concentration C, magnetic field $\mathbf{H}(0,0,H)$ and filter velocity (zero initially). Let κ, κ', α, α', $\beta(=|dt/dz|)$ and $\beta'(=|dC/dz|)$ stand for thermal diffusivity, solute diffusivity, thermal coefficient of expansion, an analogous solvent expansion, uniform temperature gradient and uniform solute gradient, respectively. Then the linearized thermosolutal hydromagnetic perturbed equations of flow through porous medium (2.6)–(2.9), (2.11) and (2.12), following the Boussinesq approximation, become

$$\frac{\varrho_0}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \mathbf{v}}{\partial t} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p + \mathbf{g}\delta\varrho + \frac{\mu_e}{4\pi}(\nabla\times\mathbf{h})\times\mathbf{H}\right] - \frac{\varrho_0 \nu}{k_1}\mathbf{v}, \tag{2.13}$$

$$\nabla\cdot\mathbf{v} = 0, \tag{2.14}$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \tag{2.15}$$

$$E'\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma, \tag{2.16}$$

$$\varepsilon\frac{\partial \mathbf{h}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{H}) + \varepsilon\eta\nabla^2\mathbf{h} - \frac{c\varepsilon}{4\pi Ne}\nabla\times[(\nabla\times\mathbf{h})\times\mathbf{H}], \tag{2.17}$$

$$\nabla\cdot\mathbf{h} = 0, \tag{2.18}$$

where $E = \varepsilon + (1-\varepsilon)[(\varrho_s c_s)/(\varrho_0 c_f)]$ and ϱ_0, c_f; ϱ_s, c_s stand for density and heat capacity of fluid and solid matrix, respectively. E' is an analogous solute parameter. c, η, N and e stand for speed of light, electrical resistivity, electron number density and charge of an electron, respectively. The equation of state is

$$\varrho = \varrho_0\left[1 - \alpha(T-T_0) + \alpha'(C-C_0)\right], \tag{2.19}$$

where the suffix zero refers to values at the reference level $z = 0$, e.g. ϱ_0, T_0 and C_0 stand for density, temperature and solute concentration at the lower boundary $z = 0$.

The change in density $\delta\varrho$, caused by the perturbations θ, γ in temperature and solute concentration, is given by

$$\delta\varrho = -\varrho_0(\alpha\theta - \alpha'\gamma). \tag{2.20}$$

Equations (2.13)–(2.18), using (2.20), give

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t}\nabla^2 w - g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\alpha\theta-\alpha'\gamma) - \frac{\mu_e H}{4\pi\varrho_0}\nabla^2\frac{\partial h_z}{\partial z}\right] = -\frac{\nu}{k_1}\nabla^2 w, \tag{2.21}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\frac{\partial\zeta}{\partial t} - \frac{\mu_e H}{4\pi\varrho_0}\frac{\partial\xi}{\partial z}\right] = -\frac{\nu}{k_1}\zeta, \tag{2.22}$$

$$\varepsilon\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z} - \frac{cH\varepsilon}{4\pi Ne}\frac{\partial\xi}{\partial z}, \tag{2.23}$$

$$\varepsilon\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)\xi = H\frac{\partial\zeta}{\partial z} + \frac{cH\varepsilon}{4\pi Ne}\nabla^2\frac{\partial h_z}{\partial z}, \tag{2.24}$$

$$\left(E\frac{\partial}{\partial t} - \kappa\nabla^2\right)\theta = \beta w, \tag{2.25}$$

$$\left(E'\frac{\partial}{\partial t} - \kappa'\nabla^2\right)\gamma = \beta' w, \tag{2.26}$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ denote the z-components of vorticity and current density, respectively.

The fluid is confined between the planes $z = 0$ and $z = d$ maintained at constant temperatures and solute concentrations. Since no perturbations in temperature and concentration are allowed and since normal component of the velocity must vanish on these surfaces, we have

$$w = 0, \quad \theta = 0 \quad \text{and} \quad \gamma = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = d. \tag{2.27}$$

Here we consider the case of two free boundaries, and the medium adjoining the fluid is electrically non-conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres (SPIEGEL [9]) and in certain geophysical situations where it is most appropriate, but it allows for an analytical solution. The condition of vanishing of tangential stresses at free surfaces implies

$$\frac{\partial^2 w}{\partial z^2} = 0 \quad \text{and} \quad \frac{\partial \zeta}{\partial z} = 0 \quad \text{at } z = 0 \text{ and } z = d. \tag{2.28}$$

Moreover,

$$\xi = (\nabla \times \mathbf{h})_z = 0 \quad \text{and } \mathbf{h} \text{ is continuous at } z = 0 \text{ and } z = d. \tag{2.29}$$

3. The dispersion relation

Here we assume the perturbations to be of the form

$$[w, \theta, \gamma, h_z, \zeta, \xi] = [W(z), \Theta(z), \Gamma(z), K(z), Z(z), X(z)] \cdot \exp(ik_x x + ik_y y + nt), \tag{3.1}$$

where k_x, k_y are horizontal wave numbers, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n is a complex constant.

Using the dimensionless variables

$$a = kd, \qquad \sigma = \frac{nd^2}{\nu}, \qquad p_1 = \frac{\nu}{\kappa}, \qquad p_2 = \frac{\nu}{\eta},$$

$$q = \frac{\nu}{\kappa'}, \qquad P_l = \frac{k_1}{d^2}, \qquad x' = \frac{x}{d}, \qquad y' = \frac{y}{d}, \qquad z' = \frac{z}{d}, \qquad \text{and} \qquad D = d/dz,$$

and removing the dashes for convenience, Eqs. (2.21)–(2.26), with the help of (3.1), become

$$\left[\frac{\sigma}{\varepsilon}(1+F\sigma)+\frac{1}{P_l}\right](D^2-a^2)W+(1+F\sigma)\frac{gd^2a^2}{\nu}(\alpha\Theta-\alpha'\Gamma) -(1+F\sigma)\frac{\mu_e Hd}{4\pi\varrho_0\nu}(D^2-a^2)DK=0, \tag{3.2}$$

$$\left[\frac{\sigma}{\varepsilon}(1+F\sigma)+\frac{1}{P_l}\right]Z=(1+F\sigma)\frac{\mu_e Hd}{4\pi\varrho_0\nu}DX, \tag{3.3}$$

$$\left[D^2-a^2-p_2\sigma\right]K=-\left(\frac{Hd}{\eta\varepsilon}\right)DW+\frac{cHd}{4\pi Ne\eta}DX, \tag{3.4}$$

$$\left[D^2-a^2-p_2\sigma\right]X=-\left(\frac{Hd}{\eta\varepsilon}\right)DZ-\frac{cH}{4\pi Ne\eta d}(D^2-a^2)DK, \tag{3.5}$$

$$\left[D^2-a^2-Ep_1\sigma\right]\Theta=-\left(\frac{\beta d^2}{\kappa}\right)W, \tag{3.6}$$

$$\left[D^2-a^2-E'q\sigma\right]\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)W. \tag{3.7}$$

The boundary conditions (2.27)–(2.29), using expression (3.1), become

$$W=D^2W=0,\quad \Theta=0,\quad \Gamma=0,\quad DZ=0,\quad X=0 \tag{3.8}$$

and h_x, h_y, h_z are continuous at $z=0,1$.

Eliminating Θ, Z, Γ, X and K between Eqs. (3.2)–(3.7), we obtain

$$\frac{\left\{\frac{\sigma}{\varepsilon}(1+F\sigma)+\frac{1}{P_l}\right\}^2}{(1+F\sigma)}\Big[(D^2-a^2)(D^2-a^2-Ep_1\sigma) \cdot(D^2-a^2-E'q\sigma)(D^2-a^2-p_2\sigma)^2\Big]W + \frac{Q}{\varepsilon}\left[\Big\{(D^2-a^2-Ep_1\sigma)(D^2-a^2-E'q\sigma)(D^2-a^2)\Big\} \cdot\left\{2\left(\frac{\sigma}{\varepsilon}\overline{1+F\sigma}+\frac{1}{P_l}\right)(D^2-a^2-p_2\sigma)+\frac{Q}{\varepsilon}(1+F\sigma)D^2\right\}\right]D^2W + \frac{M\left(\frac{\sigma}{\varepsilon}\overline{1+F\sigma}+\frac{1}{P_l}\right)^2}{(1+F\sigma)}\Big[(D^2-a^2)^2(D^2-a^2-Ep_1\sigma) \cdot(D^2-a^2-E'q\sigma)\Big]D^2W-\Big[\Big\{Ra^2(D^2-a^2-E'q\sigma) \tag{3.9}$$

(3.9) [cont.]

$$- Sa^2(D^2 - a^2 - Ep_1\sigma)\Big\}\Big\{\Big(\frac{\sigma}{\varepsilon}\overline{1 + F\sigma} + \frac{1}{P_l}\Big) \cdot (D^2 - a^2 - p_2\sigma)^2 + \frac{Q}{\varepsilon}(1 + F\sigma)(D^2 - a^2 - p_2\sigma)D^2 + M\Big(\frac{\sigma}{\varepsilon}\overline{1 + F\sigma} + \frac{1}{P_l}\Big)(D^2 - a^2)D^2\Big\}\Big]W = 0.$$

Here

$$R = \frac{g\alpha\beta d^4}{\nu\kappa}$$

is the thermal Rayleigh number,

$$S = \frac{g\alpha'\beta' d^4}{\nu\kappa}$$

is the analogous solute Rayleigh number,

$$Q = \frac{\mu_e H^2 d^2}{4\pi\varrho_0\nu\eta}$$

is the Chandrasekhar number, and

$$M = \left(\frac{cH}{4\pi N e\eta}\right)^2$$

is a non-dimensional number according to the Hall currents.

Using the boundary conditions (3.8), it can be shown with the help of Eqs. (3.2)–(3.7) that all the even-order derivatives of W vanish at the boundaries, and hence the proper solution of Eq. (3.9) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{3.10}$$

where W_0 is a constant. Substituting (3.10) in Eq. (3.9) and letting

$$a^2 = \pi^2 x, \qquad R_1 = \frac{R}{\pi^4}, \qquad S_1 = \frac{S}{\pi^4}, \qquad Q_1 = \frac{Q}{\pi^2},$$

$$i\sigma_1 = \frac{\sigma}{\pi^2} \qquad \text{and} \qquad P = \pi^2 P_l,$$

we obtain the dispersion relation

$$R_1 x = \Big[\frac{(1 + x)(1 + x + iEp_1\sigma_1)}{(1 + i\pi^2 F\sigma_1)}\Big(\frac{i\sigma_1}{\varepsilon}\overline{1 + i\pi^2 F\sigma_1} + \frac{1}{P}\Big)^2 \Big\{(1 + x + ip_2\sigma_1)^2 + M(1 + x)\Big\} + \frac{Q_1}{\varepsilon}(1 + x)(1 + x + iEp_1\sigma_1) \tag{3.11}$$

(3.11) [cont.]

$$\left\{2(1+x+ip_2\sigma_1)\left(\frac{i\sigma_1}{\varepsilon}\overline{1+i\pi^2F\sigma_1}+\frac{1}{P}\right)+\frac{Q_1}{\varepsilon}(1+i\pi^2F\sigma_1)\right\}\Bigg] \Bigg/ \Bigg[\left(\frac{i\sigma_1}{\varepsilon}\overline{1+i\pi^2F\sigma_1}+\frac{1}{P}\right)\left\{(1+x+ip_2\sigma_1)^2+M(1+x)\right\} + \frac{Q_1}{\varepsilon}(1+i\pi^2F\sigma_1)(1+x+ip_2\sigma_1) + S_1x\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q\sigma_1)}\Bigg].$$

4. The stationary convection

For stationary convection, $\sigma = 0$ and Eq. (3.11) reduces to

$$R_1 = \left(\frac{1+x}{x'}\right)\frac{\left(\dfrac{1+x}{P}+\dfrac{Q_1}{\varepsilon}\right)^2+\dfrac{M(1+x)}{P^2}}{\dfrac{1+x}{P}+\dfrac{Q_1}{\varepsilon}+\dfrac{M}{P}}+S_1, \tag{4.1}$$

and the Maxwellian viscoelastic fluid behaves like an ordinary Newtonian fluid. In order to investigate the effects of Hall current, stable solute gradient and medium permeability, we examine the behaviour of dR_1/dM, dR_1/dS_1 and dR_1/dP analytically.

Equation (4.1) yields

$$\frac{dR_1}{dM} = -\left(\frac{1+x}{\varepsilon x}\right)Q_1\frac{\left(\dfrac{1+x}{P}+\dfrac{Q_1}{\varepsilon}\right)}{\left(\dfrac{1+x+M}{P}+\dfrac{Q_1}{\varepsilon}\right)^2}, \tag{4.2}$$

which is negative. The Hall current, therefore, has a destabilizing effect on the thermosolutal convection in porous medium. It is evident from Eq. (4.1) that

$$\frac{dR_1}{S_1} = +1, \tag{4.3}$$

implying threby that stable solute gradient has a stabilizing effect on the thermosolutal convection in porous medium.

Equation (4.1) also yields

$$\frac{dR_1}{dP} = -\frac{(1+x)}{xP^2}\frac{\dfrac{(1+x)}{P^2}(1+x+M)^2+\dfrac{2Q_1(1+x)}{\varepsilon P}+\dfrac{Q_1^2}{\varepsilon^2}(1+x-M)}{\left(\dfrac{1+x+M}{P}+\dfrac{Q_1}{\varepsilon}\right)^2}, \tag{4.4}$$

which is negative if $i + x > M$. The condition $i + x > M$ is met for all wave numbers as the Hall current parameter $M \ll 1$. The medium permeability, therefore, has a destabilizing effect on thermosolutal convection in porous medium in a Maxwellian viscoelastic fluid for the stationary convection.

5. The overstable case

Here we discuss the possibility as to whether instability may occur as overstability. Equating real and imaginary parts of Eq. (3.11) and eliminating R_1 between them, we obtain

$$A_6c_1^6 + A_5c_1^5 + A_4c_1^4 + A_3c_1^3 + A_2c_1^2 + A_1c_1 + A_0 = 0, \tag{5.1}$$

where we have written $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_6 = -\frac{\pi^6F^3p_2^4E'qb}{\varepsilon^3}\left[2EE'p_1q + \pi^2Fb(Ep_1 - E'q)\right], \tag{5.2}$$

$$\begin{aligned} A_0 = {} & \frac{1}{P^2}\left(\frac{1}{\varepsilon} - \frac{\pi^2F}{P}\right)b^8 + \left[\frac{Ep_1}{P^3} + \frac{2M}{\varepsilon P^2} + \frac{2\pi^2FQ_1}{\varepsilon P^2}(b-1)\right. \\ & \left. + \frac{2Q_1}{\varepsilon p}\left(\frac{1}{\varepsilon} - \frac{\pi^2F}{P}\right)\right]b^7 + \left[\frac{2Q_1^2\pi^2F(b-1)}{\varepsilon^2P} + \frac{2MEp_1}{P^3}\right. \\ & + \frac{Q_1}{\varepsilon P^2}(Ep_1 - p_2) + \frac{2Q_1}{\varepsilon P^2}\left(Ep_1 - \pi^2FM\right) + \left(\frac{M}{P} + \frac{Q_1}{\varepsilon}\right)^2 \\ & \left.\left(\frac{1}{\varepsilon} - \frac{\pi^2F}{P}\right)\right]b^6 + \left[\frac{MQ_1}{\varepsilon P^2}(p_2 + 3Ep_1) + \frac{M^2Ep_1}{P^3}\right. \\ & \left. + \frac{Q_1^2}{\varepsilon^2}\left\{\frac{2}{P}(Ep_1 - p_2) + \frac{\pi^2FM}{P} + \left(\frac{Ep_1}{P} - \frac{M}{\varepsilon}\right)\right\}\right]b^5 \\ & + \frac{Q_1^2}{\varepsilon^2}\left[\frac{Q_1}{\varepsilon}(Ep_1 - p_2) + \frac{MEp_1}{P}\right]b^4 + S_1(b-1)\left(\frac{M+b}{P}\right) \\ & (Ep_1 - E'q)\left\{\frac{b}{P} + \frac{Q_1}{\varepsilon} + \frac{M}{P}\right\}b^3. \end{aligned} \tag{5.3}$$

The six values of c_1, σ_1 being real, are positive. The product of the roots $(= A_0/A_6)$ is positive.

A_6 is negative if

$$Ep_1 > E'q, \tag{5.4}$$

and A_0 is positive if

$$Ep_1 > p_2, \quad Ep_1 > E'q, \quad \frac{1}{\varepsilon} > \frac{\pi^2F}{P}, \quad Ep_1 > M\pi^2F \quad \text{and} \quad \frac{Ep_1}{P} > \frac{M}{\varepsilon}. \tag{5.5}$$

The inequalities (5.4) and (5.5) imply that the sufficient conditions for non-existence of overstability are

$$Ep_1 > E'q, \qquad \frac{1}{\varepsilon} > \frac{\pi^2 F}{P}$$

and

$$Ep_1 > \text{maximum of } \left(p_2, \frac{MP}{\varepsilon}\right),$$

i.e

$$E'\kappa < E\kappa', \qquad \lambda < \frac{k_1}{\nu\varepsilon}$$

and

$$\frac{E\nu}{\kappa} > \text{maximum of } \left[\frac{\nu}{\eta}, \left(\frac{\pi c H}{4\pi N e}\right)^2 \frac{k_1}{\varepsilon d^2}\right],$$

$$E'\kappa < E\kappa', \qquad \lambda < \frac{k_1}{\nu\varepsilon}$$

and

$$\frac{E\nu}{\kappa} > \text{maximum of } \left[\frac{\nu}{\eta}, \left(\frac{\pi c H}{4\pi N e}\right)^2 \frac{k_1}{\varepsilon d^2}\right].$$

These are, therefore, the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

6. Conclusions

A Maxwellian viscoelastic fluid layer heated and soluted from below in a porous medium is considered to include the effect of the Hall currents. For stationary convection, the Maxwellian viscelastic fluid behaves like an ordinary Newtonian fluid and stable solute gradient is found to postpone the onset of instability, whereas medium permeability and Hall currents speed up the onset of instability. The sufficient conditions for the non-existence of overstability are obtained, the violation of which does no necessarily imply the occurrence of overstability. The problem and the results have relevance and importance for geophysics.

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