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Transient free convection flow of dissipative fluid past an infinite vertical porous plate

V. M. SOUNDALGEKAR(1), A. G. UPLEKAR(2), B. S. JAISWAL(3)

(1) *31A-12, Brindavan Society*
Thane (400 601) India

(2) *Patkar College, Goregaon*
Bombay (400 062) India

(3) *Birla College*
Kalyan (421 304) India

A finite-difference analysis of transient free convection flow of a dissipative fluid past an infinite vertical porous plate is presented here. Velocity, temperature, skin-friction and Nusselt number are shown graphically and the effects of different parameters on the flow field are discussed.

1. Introduction

In 60's, many papers were published on the transient free convection flow of a viscous incompressible fluid past an infinite vertical plate under different physical conditions like isothermal plate, constant heat flux at the plate etc.

Illingworth [1] was the first to study the transient free convection flow past an infinite vertical isothermal plate by assuming the Prandtl number as unity. Later on, Siegel [2] studied the transient free convection flow past a semi-infinite vertical plate by momentum integral method for a step-change in surface temperature and surface heat flux and he pointed out that the initial behaviour of the temperature and velocity fields for a semi-infinite vertical plate is the same as for the doubly infinite vertical flat plate. The temperature field is given by the solution of an unsteady one-dimensional heat conduction problem and the transition to convection begins only when some effect from the leading edge has propagated up the plate to a particular point. Before this time, the fluid in this region effectively does not know that the plate has a leading edge.

Later on, there were many papers published on this topic viz. those by Schetz and Eichhorn [3], Menold and Yang [4[, Goldstein and Briggs [5], Soundalgekar [6–10] for infinite plate and for a semi-infinite plate, this problem was studied by Sparrow and Gregg [11]. Chung and Anderson [12], Hellums and Churchill [13, 14] by boundary layer method.

In all these papers, however, the effect of viscous dissipation was assumed to be negligible. But it was GEBHART [15] who first showed that in steady free-convection flow, the viscous dissipation heat cannot be neglected for fluids with high Prandtl number or flow at high gravitational field – or in rotational flow. Hence it is essential to know the effects of viscous dissipative heat on the transient free convective flow past an infinite vertical plate with a step-change in temperature. So SOUNDALGEKAR *et al.* [16] studied the effects of viscous dissipative heat on the transient free convection flow past an infinite vertical plate with a step-change in plate-temperature. The problem governed by nonlinear coupled partial differential equations was solved by finite-difference method.

Now, the study of flow of viscous fluid past vertical porous plate is also of importance in industrial applications. Free convection flow past a semi-infinite vertical porous plate was studied by many by considering both suction and injection. Notable amongst them are by NANDA and SHARMA [17], SINGH [18], who studied the effect of suction on the transient free convection flow past a vertical porous plate. In all these studies, the viscous dissipative heat was neglected. On taking into account viscous dissipative heat, how transient free convection flow past a porous plate is affected by suction? This has not been studied in the literature. Hence, the motivation. In Sec. 2, the mathematical analysis is presented by including viscous dissipative effects in the energy equation and in Sec. 3, the conclusions are set out.

2. Mathematical analysis

Consider an infinite vertical isothermal porous plate surrounded on one side by infinite mass of fluid like air or water and both at same temperature T'_∞ initially. At time $t' > 0$, the plate temperature is raised to T'_w, causing the presence of temperature difference $T'_w - T'_\infty$. As the plate is infinite in extent, the physical variables are functions of y' and t' where y' is taken normal to the plate and the x'-axis is taken along the plate in the vertically upward direction. Then the flow can be shown to be governed, under usual Boussinesq's approximation, by the following equations:

$$\frac{\partial v'}{\partial y'} = 0, \tag{2.1}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta \left[T' - T'_\infty\right], \tag{2.2}$$

$$\rho \; C_p \left[\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'}\right] = k \frac{\partial^2 T'}{\partial y'^2} + \mu \left[\frac{\partial u'}{\partial y'}\right]^2 \tag{2.3}$$

with the following initial and boundary conditions:

$$t' \le 0, \quad u' = 0, \quad T' \to T'_\infty \quad \text{for all } y', \tag{2.4}$$
$$t > 0, \quad u' = 0, \quad T' = T'_w \quad \text{as } y' = 0,$$
$$u' = 0, \quad T' \to T'_\infty \quad \text{as } y' \to \infty.$$

Here u', v' are the velocity components along and normal to the plate, ν the kinematic viscosity, g the acceleration due to gravity, β the volumetric coefficient of thermal expansion, ρ the density, C_p the specific heat at constant pressure, k the thermal conductivity and μ is the viscosity of the fluid.

On introducing following non-dimensional quantities

$$u = u'/u_0, \quad y = y'/L, \quad t = t'/t_0, \quad \Delta T = T'_w - T'_\infty,$$
$$\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \Pr = \frac{\mu\, C_p}{k}, \quad E = \frac{u_0^2}{C_p \Delta T}, \quad \gamma = v_0/u_0, \tag{2.5}$$

where

$$u_0 = [\nu\, g\beta\, \Delta T]^{1/3}, \quad L = \left[g\beta\, \Delta T/\nu^{2}\right]^{-1/3}, \quad t_o = [g\beta\, \Delta T]^{-2/3}/\nu^{-1/3}$$

in Eqs. (2.1) to (2.4), we have

$$\frac{\partial u}{\partial t} - \gamma \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta, \tag{2.6}$$

$$\Pr\left[\frac{\partial \theta}{\partial t} - \gamma \frac{\partial \theta}{\partial y}\right] = \frac{\partial^2 \theta}{\partial y^2} + \Pr \mathrm{E}\left[\frac{\partial u}{\partial y}\right]^2 \tag{2.7}$$

with the following initial and boundary conditions

$$u = 0, \quad \theta = 0, \quad \text{for all } y, \quad t \le 0, \tag{2.8}$$
$$u = 0, \quad \theta = 1, \quad \text{at } y = 0,$$
$$u = 0, \quad \theta = 0, \quad \text{as } y \to \infty.$$

Here Pr is the Prandtl number, E, the Eckert number and γ is the suction parameter. Also u_0, L and t_0 are reference velocity, length and time respectively. These equations (2.6) to (2.8) are now solved by implicit finite difference schemes of Crank–Nicolson type. The finite difference approximations to these equations are as follows:

$$\Pr\left[\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} - \gamma \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y}\right] = \frac{1}{2}\left[\frac{\theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{(\Delta y)^2} + \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta y)^2}\right] + \Pr \mathrm{E}\left[\frac{u_{i+1,j} - u_{i,j}}{\Delta y}\right]^2, \tag{2.9}$$

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}-\gamma\,\frac{u_{i+1,j}-u_{i,j}}{\Delta y}=\frac{1}{2}\left[\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}}{(\Delta y)^2}+\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2}\right]+\frac{1}{2}\left[\theta_{\,i,j+1}+\theta\, i,j\right]. \tag{2.10}$$

The initial and boundary conditions take the following forms

$$\begin{aligned} &U_{i,0}=0, \quad \theta_{\,i,0}=0 \quad \text{for all } i \text{ except } i=0,\\ &U_{0,j}=0, \quad \theta_{\,0,j}=1,\\ &U_{M,j}=0, \quad \theta_{\,M,j}=0, \end{aligned} \tag{2.11}$$

where M corresponds to ∞.

We now write Eqs. (2.9) and (2.10) in the following form:

$$A_1\theta_{\,i-1,j+1}+B_1\theta_{\,i,j+1}+D_1\theta_{\,i+1,j+1}=E_1, \tag{2.12}$$

$$A_2U_{i-1,j+1}+B_2U_{i,j+1}+D_2U_{i+1,j+1}=E_2, \tag{2.13}$$

where

$$R=\frac{\Delta t}{2(\Delta y)^2}, \quad R_1=\frac{\Delta t}{2(\Delta y)^2.\,\mathrm{Pr}},$$

$$A_1=R_1, \quad B_1=-2R_1-1, \quad D_1=R_1,$$

$$E_1=-R_1\theta_{\,i-1,j}+\Big(2R+\frac{\gamma\,\Delta t}{\Delta y}-1\Big)\theta_{\,i,j}+\left[-\frac{\gamma\,\Delta t}{\Delta y}-R\right]\theta_{\,i+1,j}-\frac{E\Delta t}{\big(\Delta y\big)^2}[\,U_{i+1,j}-U_{i,j}]^2,$$

$$A_2=R, \quad B_2=2R-1, \quad D_2=R,$$

$$E_2=-R\,U_{i-1,j}+\Big(2R+\frac{\gamma\,\Delta t}{\Delta\,y}-1\Big)U_{i,j}+\Big(-\frac{\gamma\,\Delta t}{\Delta\,y}-R\Big)U_{i+1,j}-\frac{\Delta t}{2}\Big(\theta_{\,i,j+1}+\theta_{\,i,j}\Big).$$

Here the suffix i corresponds to y and j corresponds to t. Also $\Delta t=t_{j+1}-t_j$ and $\Delta y=y_{i+1}-y_i$.

3. Solutions

Knowing the values of θ, u at a time t, we can calculate the values at a time $t + \Delta t$ as follows. We substitute i = 1,2, ..., M − 1 in Eq. (2.12) which results in a tri-diagonal system of equations in unknown values of θ. Using initial and boundary conditions, the system can be solved by Gauss elimination method [CARNAHAN *et al.* (19)]. Thus θ is known at all values of y at time $t + \Delta t$. Then knowing the values of θ and applying the same procedure and using boundary conditions, we calculate u from (2.13). This procedure is continued to obtain the solution till desired time t. Computations are carried out for air (Pr = 0.71) and water (Pr = 7.0). In order to check the accuracy, these results are computed with usual explicit finite difference technique and the results computed from both explicit and implicit method are found to agree well.

We observe from Fig. 1 and 2 that greater viscous dissipative heat causes a rise in the transient velocity of air, but an increase in suction parameter γ leads to a decrease in the transient velocity. The transient velocity also decreases with increasing the Prandtl number but increases with increasing time t. We have computed the velocity for values of time t such that there is no change in the value of velocity which we call it as steady-state velocity. It is observed that the time necessary to reach this steady-state is $t_s = 8.5$, where t_s is the steady-state value of time t. The temperature profiles are shown on Figs. 3 and 4 and we observe from these figures that greater viscous dissipative heat causes a rise in the temperature. However, from Fig. 4, as compared to the temperature at $E = 0$, the temperature in the presence of viscous dissipative heat is always less. From Fig. 4, we observe that the temperature in the presence of a permeable plate is more than that in the presence of an impermeable plate. The transient temperature is also found to decrease with increasing the Prandtl number of the fluid, but increases with increasing time t. The steady-state temperature u reached when $t = t_s = 8.5$.

From the velocity field, we now study the skin friction. It is given by

$$\tau' = -\mu \left.\frac{\partial u'}{\partial y'}\right|_{y=0} \tag{3.1}$$

and in view of (2.5), Eq. (3.1) reduces to

$$\tau = -\left.\frac{\partial u}{\partial y}\right|_{y=0}, \tag{3.2}$$

where $\tau = \tau'/\rho\, u_0^2$.

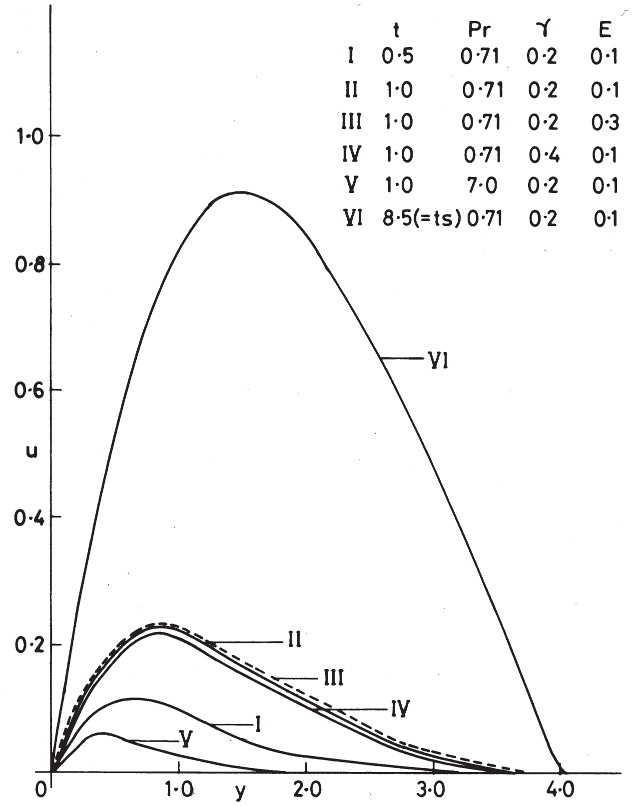

Fig. 1. Velocity profiles.

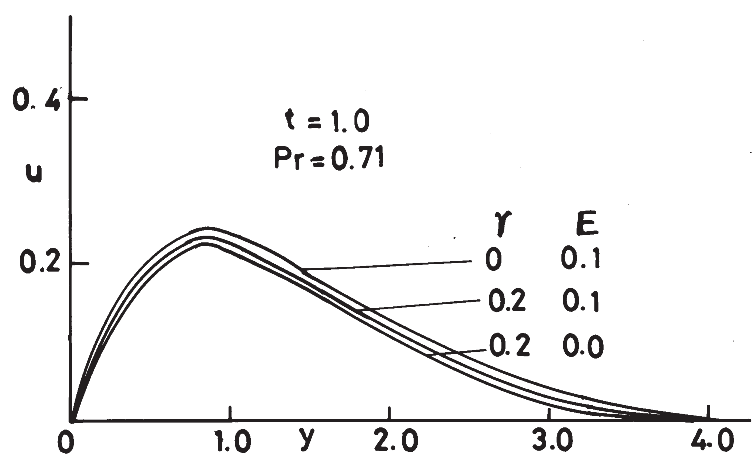

Fig. 2. Velocity profiles.

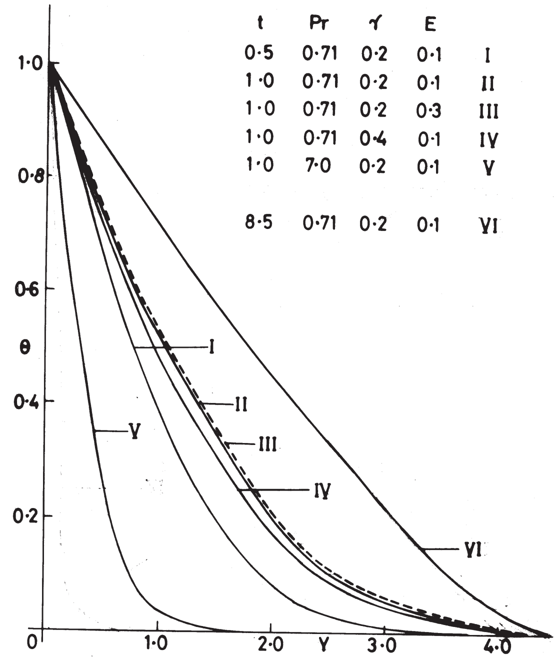

Fig. 3. Temperature profiles.

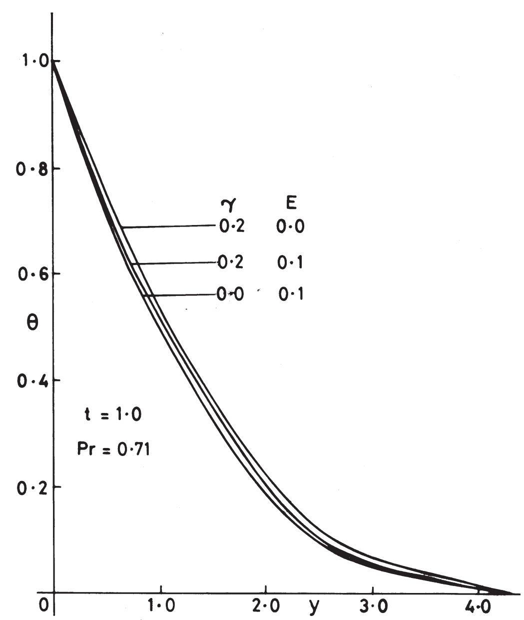

Fig. 4. Temperature profiles.

The value of the derivative is computed by using Newton–Coates formula for five points these are shown in Figs. 5 and 6. We observe from these figures that greater viscous dissipative heat causes a rise in the skin friction of air but there is a reduction in the skin friction owing to an increase in the suction parameter γ. The skin friction also decreases with increasing the Prandtl number. The skin friction increases with increasing time t when $t < 4.0$, but at large values of time t, it is not significantly affected by time t. From Fig. 6, we observe that, as compared to $E = 0$, the skin friction is greater when viscous dissipative heat is present. Also, in the presence of an impermeable plate, the skin friction is less as compared to that in the presence of a permeable plate.

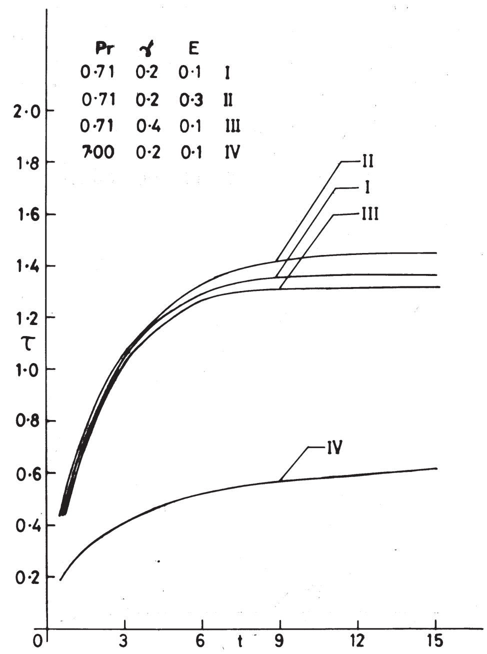

Fig. 5. Skin friction.

The rate of heat transfer is given by

$$q = -k \left.\frac{\partial T'}{\partial y'}\right|_{y'=0} \tag{3.3}$$

and in view of (2.5), Eq. (3.3) reduces to

$$\mathrm{Nu} = \frac{qL}{k\Delta T} = -\left.\frac{\partial \theta}{\partial y}\right|_{y=0}. \tag{3.4}$$

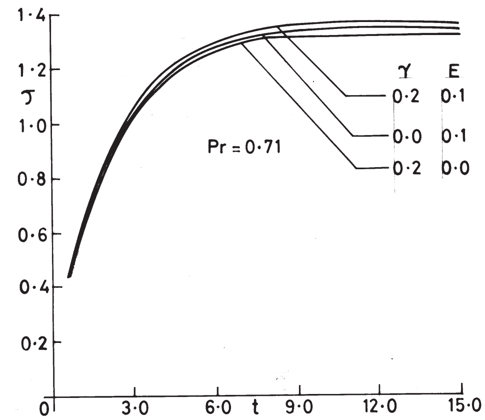

FIG. 6. Skin friction.

The numerical values of Nu are computed as in case of the skin friction and these are shown in Fig. 7. The Nusselt number is observed to decrease owing to greater viscous dissipative heat but increases when the suction parameter γ increases. It also increases with increasing the Prandtl number of the fluid. The Nusselt number is found to decrease with time, when $t \leq 3.0$ and remains unaffected by time t when $t \geq 3.0$.

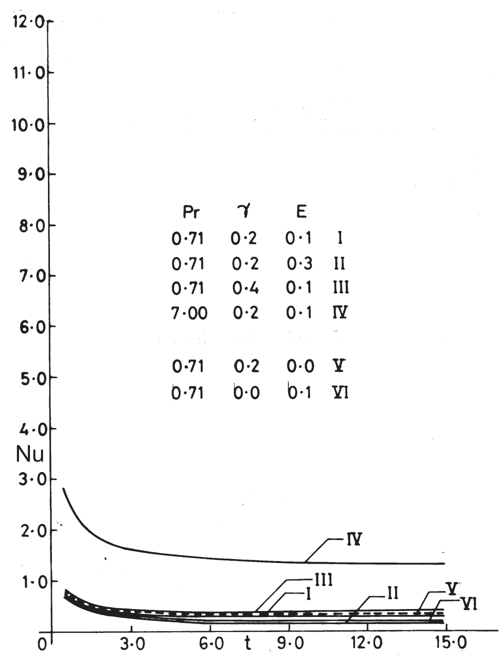

FIG. 7. Nusselt number.

4. Conclusions

1. There is a rise in the velocity and temperature of air owing to greater viscous dissipative heat.
2. Owing to increase in the suction velocity, there is a decrease in the velocity and a fall in the temperature of air.
3. With increase in the Prandtl number, there is always a fall in the velocity and temperature.
4. Owing to greater viscous dissipative heat, there is a rise in the skin friction and a fall in the Nusselt number.
5. Skin-friction decreases and the Nusselt number increases due to increasing the suction parameter.

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